Resumen de fórmulas de coordenadas curvilíneas ortogonales

$x = \rho\cos\varphi = r\,\mathrm{sen}\,\theta\cos\varphi$	$\sqrt{x^2+y^2} = \rho = r\,\mathrm{sen}\,\theta$	$\sqrt{x^2+y^2+z^2} = \sqrt{\rho^2+z^2} = r$
$y = \rho\,\mathrm{sen}\,\varphi = r\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi$	$\mathrm{arctg}\,\dfrac{y}{x} = \varphi = \varphi$	$\mathrm{arctg}\,\dfrac{\sqrt{x^2+y^2}}{z} = \mathrm{arctg}\,\dfrac{\rho}{z} = \theta$
$z = z = r\cos\theta$	$z = z = r\cos\theta$	$\mathrm{arctg}\,\dfrac{y}{x} = \varphi = \varphi$

Vector de posición:

$\mathbf{r} = x\mathbf{u}_x + y\,\mathbf{u}_y + z\mathbf{u}_z$
$\mathbf{r} = \rho\,\mathbf{u}_\rho + z\mathbf{u}_z$
$\mathbf{r} = r\mathbf{u}_r$

Factores de escala:

$$h_i = \left|\frac{\partial \mathbf{r}}{\partial q_i}\right|$$

$h_x = 1$	$h_y = 1$	$h_z = 1$
$h_\rho = 1$	$h_\varphi = \rho$	$h_z = 1$
$h_r = 1$	$h_\theta = r$	$h_\varphi = r\,\mathrm{sen}\,\theta$

Diferencial de volumen:

$$d\tau = h_1 h_2 h_3\, dq_1\, dq_2\, dq_3$$

$d\tau = dx\, dy\, dz$
$d\tau = \rho\, d\rho\, d\varphi\, dz$
$d\tau = r^2\,\mathrm{sen}\,\theta\, dr\, d\theta\, d\varphi$

Diferencial de longitud:

$d\mathbf{r} = h_1\, dq_1\, \mathbf{u}_1 + h_2\, dq_2\, \mathbf{u}_2 + h_3\, dq_3\, \mathbf{u}_3$
$d\mathbf{r} = dx\mathbf{u}_x + dy\,\mathbf{u}_y + dz\mathbf{u}_z$
$d\mathbf{r} = d\rho\,\mathbf{u}_\rho + \rho\, d\varphi\,\mathbf{u}_\varphi + dz\mathbf{u}_z$
$d\mathbf{r} = dr\mathbf{u}_r + r\, d\theta\,\mathbf{u}_\theta + r\,\mathrm{sen}\,\theta\, d\varphi\,\mathbf{u}_\varphi$

Diferencial de superficie coordenada:

$$d\mathbf{S}\big|_{q_3=\mathrm{cte}} = h_1 h_2 dq_1 dq_2 \mathbf{u}_3$$

$d\mathbf{S}_x = dy\, dz\, \mathbf{u}_x$	$d\mathbf{S}_y = dx\, dz\, \mathbf{u}_y$	$d\mathbf{S}_z = dx\, dy\, \mathbf{u}_z$
$d\mathbf{S}_\rho = \rho\, d\varphi\, dz\, \mathbf{u}_\rho$	$d\mathbf{S}_\varphi = d\rho\, dz\, \mathbf{u}_\varphi$	$d\mathbf{S}_z = \rho\, d\rho\, d\varphi\, \mathbf{u}_z$
$d\mathbf{S}_r = r^2\ \mathrm{sen}\,\theta\, d\theta\, d\varphi\, \mathbf{u}_r$	$d\mathbf{S}_\theta = r\ \mathrm{sen}\,\theta\, dr\, d\varphi\, \mathbf{u}_\theta$	$d\mathbf{S}_\varphi = r\ dr\, d\theta\, \mathbf{u}_\varphi$

Gradiente:

$\nabla\phi = \dfrac{1}{h_1}\dfrac{\partial\phi}{\partial q_1}\mathbf{u}_1 + \dfrac{1}{h_2}\dfrac{\partial\phi}{\partial q_2}\mathbf{u}_2 + \dfrac{1}{h_3}\dfrac{\partial\phi}{\partial q_3}\mathbf{u}_3$
$\nabla\phi = \dfrac{\partial\phi}{\partial x}\mathbf{u}_x + \dfrac{\partial\phi}{\partial y}\mathbf{u}_y + \dfrac{\partial\phi}{\partial z}\mathbf{u}_z$
$\nabla\phi = \dfrac{\partial\phi}{\partial \rho}\mathbf{u}_\rho + \dfrac{1}{\rho}\dfrac{\partial\phi}{\partial \varphi}\mathbf{u}_\varphi + \dfrac{\partial\phi}{\partial z}\mathbf{u}_z$
$\nabla\phi = \dfrac{\partial\phi}{\partial r}\mathbf{u}_r + \dfrac{1}{r}\dfrac{\partial\phi}{\partial \theta}\mathbf{u}_\theta + \dfrac{1}{r\,\mathrm{sen}\,\theta}\dfrac{\partial\phi}{\partial \varphi}\mathbf{u}_\varphi$

Vectores unitarios:

$$\mathbf{u}_i = \frac{1}{h_i}\frac{\partial \mathbf{r}}{\partial q_i}$$

$\cos\varphi\mathbf{u}_x + \mathrm{sen}\,\varphi\mathbf{u}_y = \mathbf{u}_\rho = \mathrm{sen}\,\theta\mathbf{u}_r + \cos\theta\mathbf{u}_\theta$	$\mathbf{u}_x = \cos\varphi\mathbf{u}_\rho - \mathrm{sen}\,\varphi\mathbf{u}_\varphi = \mathrm{sen}\,\theta\cos\varphi\mathbf{u}_r + \cos\theta\cos\varphi\mathbf{u}_\theta - \mathrm{sen}\,\varphi\mathbf{u}_\varphi$
$-\mathrm{sen}\,\varphi\mathbf{u}_x + \cos\varphi\mathbf{u}_y = \mathbf{u}_\varphi = \mathbf{u}_\varphi$	$\mathbf{u}_y = \mathrm{sen}\,\varphi\mathbf{u}_\rho + \cos\varphi\mathbf{u}_\varphi = \mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\mathbf{u}_r + \cos\theta\,\mathrm{sen}\,\varphi\mathbf{u}_\theta + \cos\varphi\mathbf{u}_\varphi$
$\mathbf{u}_z = \mathbf{u}_z = \cos\theta\mathbf{u}_r - \mathrm{sen}\,\theta\mathbf{u}_\theta$	$\mathbf{u}_z = \mathbf{u}_z = \cos\theta\mathbf{u}_r - \mathrm{sen}\,\theta\mathbf{u}_\theta$
	$\mathrm{sen}\,\theta\cos\varphi\mathbf{u}_x + \mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\mathbf{u}_y + \cos\theta\mathbf{u}_z = \mathrm{sen}\,\theta\mathbf{u}_\rho + \cos\theta\mathbf{u}_z = \mathbf{u}_r$
	$\cos\theta\cos\varphi\mathbf{u}_x + \cos\theta\,\mathrm{sen}\,\varphi\mathbf{u}_y - \mathrm{sen}\,\theta\mathbf{u}_z = \cos\theta\mathbf{u}_\rho - \mathrm{sen}\,\theta\mathbf{u}_z = \mathbf{u}_\theta$
	$-\mathrm{sen}\,\varphi\mathbf{u}_x + \cos\varphi\mathbf{u}_y = \mathbf{u}_\varphi = \mathbf{u}_\varphi$

Rotacional:

$$\nabla\times\mathbf{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\mathbf{u}_1 & h_2\mathbf{u}_2 & h_3\mathbf{u}_3 \\ \dfrac{\partial}{\partial q_1} & \dfrac{\partial}{\partial q_2} & \dfrac{\partial}{\partial q_3} \\ h_1A_1 & h_2A_2 & h_3A_3 \end{vmatrix}$$

$$\nabla\times\mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\mathbf{u}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\mathbf{u}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\mathbf{u}_z$$

$$\nabla\times\mathbf{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\mathbf{u}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\mathbf{u}_\varphi + \frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \varphi}\right)\mathbf{u}_z$$

$$\nabla\times\mathbf{A} = \frac{1}{r\,\mathrm{sen}\,\theta}\left(\frac{\partial(\mathrm{sen}\,\theta A_\varphi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \varphi}\right)\mathbf{u}_r + \frac{1}{r}\left(\frac{1}{\mathrm{sen}\,\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial(rA_\varphi)}{\partial r}\right)\mathbf{u}_\theta + \frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\mathbf{u}_\varphi$$

Divergencia:

$$\nabla\cdot\mathbf{A} = \frac{1}{h_1h_2h_3}\left(\frac{\partial(A_1h_2h_3)}{\partial q_1} + \frac{\partial(h_1A_2h_3)}{\partial q_2} + \frac{\partial(h_1h_2A_3)}{\partial q_3}\right)$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial(\mathrm{sen}\,\theta A_\theta)}{\partial \theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

Laplaciano:

$$\nabla^2\phi = \frac{1}{h_1h_2h_3}\left(\frac{\partial}{\partial q_1}\left(\frac{h_2h_3}{h_1}\frac{\partial\phi}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{h_1h_3}{h_2}\frac{\partial\phi}{\partial q_2}\right) + \frac{\partial}{\partial q_3}\left(\frac{h_1h_2}{h_3}\frac{\partial\phi}{\partial q_3}\right)\right)$$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

$$\nabla^2\phi = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial\phi}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\phi}{\partial \varphi^2} + \frac{\partial^2\phi}{\partial z^2}$$

$$\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2\,\mathrm{sen}\,\theta}\frac{\partial}{\partial \theta}\left(\mathrm{sen}\,\theta\frac{\partial\phi}{\partial \theta}\right) + \frac{1}{r^2\,\mathrm{sen}^2\,\theta}\frac{\partial^2\phi}{\partial \varphi^2}$$